The Use of the Poynting Vector in Interpreting ULF Waves

T. Elsden   A.N. Wright

University of St Andrews

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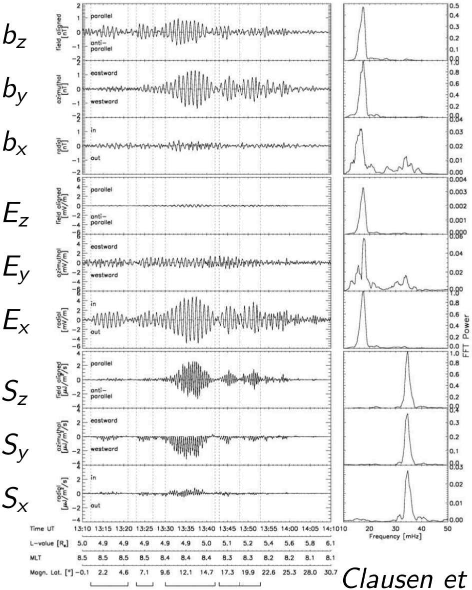
Overview

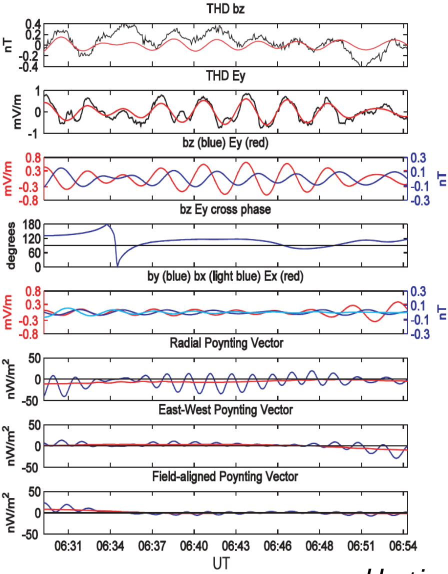
- We are interested in the numerical modeling of ULF wave phenomena in the outer dayside magnetosphere.
- Work from the theoretical foundations of magnetospheric waveguides.
- We model 2 very different observations from Cluster and THEMIS.
- What can be inferred from a simple waveguide model?

Cluster Observation



Clausen et al., [2008]

- Dominant frequency 17.2mHz - Pc4 range
- Strong b_y, b_z and E_x signals.
- Purely tailward S_y.
- Cluster location: magnetic latitude $\sim 12^\circ$, near plasmapause, dawn sector.

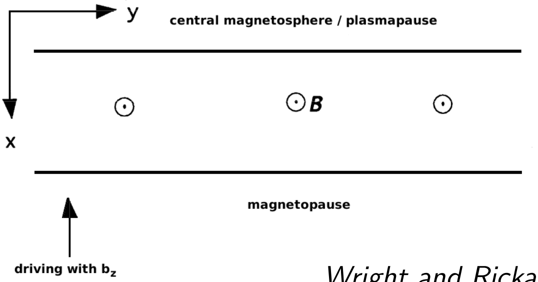


- Frequency 6.5mHz, global mode.
- Dominant b_z and E_y perturbations.
- Radially inwards S_x.
- THD location: magnetic latitude $\sim 3^\circ$, near plasmapause, dawn sector.

Hartinger et al., [2012]

Model

We model the magnetosphere using a waveguide based on the hydromagnetic box implemented by *Kivelson and Southwood*, [1986].



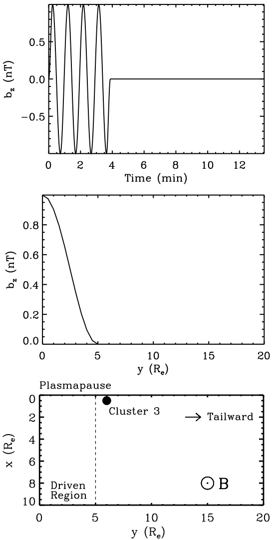
Wright and Rickard, [1995]

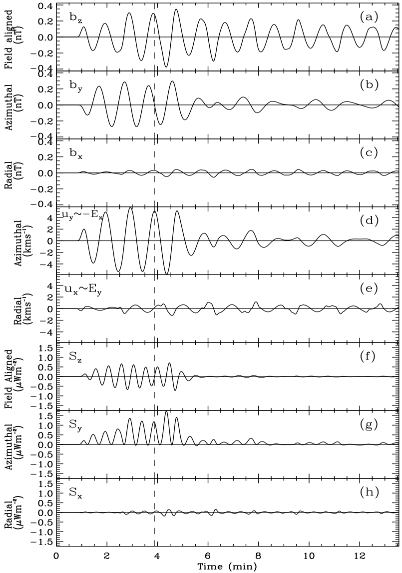
- Uniform background magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$.
- $\hat{\mathbf{x}}$ radially outwards, $\hat{\mathbf{y}}$ azimuthal coordinate.
- Let $\rho = \rho(x) \Rightarrow V_A = V_A(x)$.

Boundary Condition for Driven Boundary

- Implement a new boundary condition on the driven magnetopause boundary.
- Drive with b_z perturbation to mimick pressure driving, the main source of ULF waves [*Takahashi and Ukhorskiy*, 2008].
- Can prove this gives a node of b_z (antinode u_x) at driven boundary.
- Yields a quarter wavelength fundamental radial mode.
- Can reduce fundamental eigenfrequencies without resorting to unphysical higher plasma densities [*Mann et al.*, 1999].

Results - Cluster Simulation





b_z

b_y

b_x

$u_y \sim -E_x$

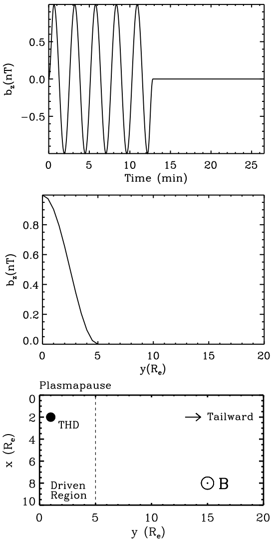
$u_x \sim E_y$

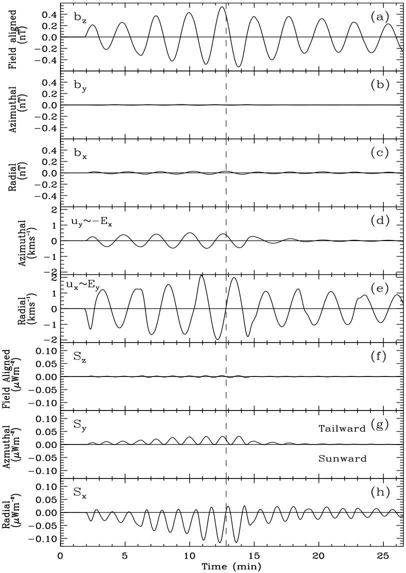
S_z

S_y

S_x

Results - THEMIS Simulation





b_z

b_y

b_x

$u_y \sim -E_x$

$u_x \sim E_y$

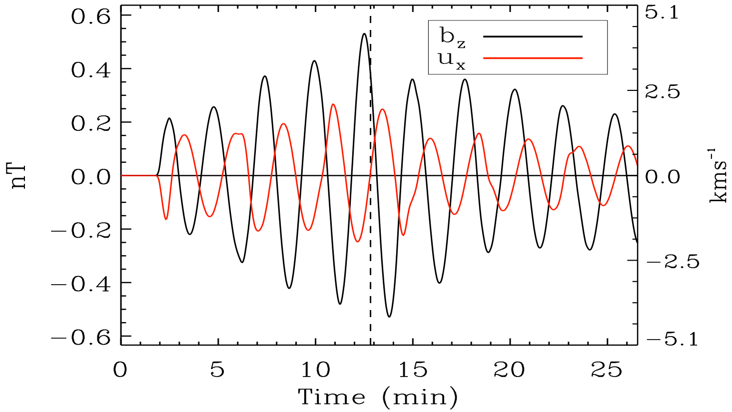
S_z

S_y

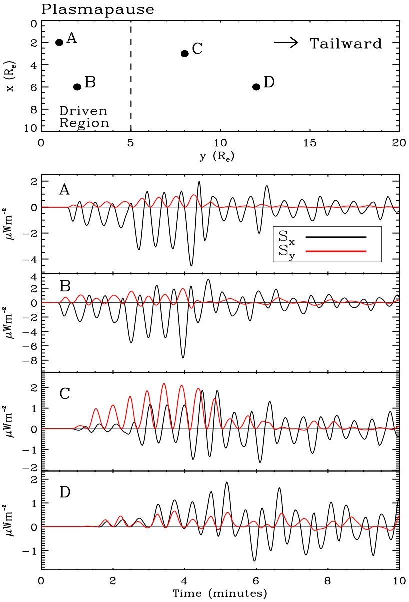
S_x

Results - THEMIS Simulation

- Difference between driving and post driving phase shifts.
- Can be used in observations to infer the end of the driving phase.



Results - Satellite Position



- Plots made using Cluster simulation parameters.
- Position A models the position of THD.
- Locations B-D display the change in S_x signature from inward to outward.
- S_y further downtail is always tailward.

Conclusions

- Have modeled two different observations from Cluster and THEMIS.
- Our simple simulation could match to the main features of the observations.
- Developed a new boundary condition to effectively drive with pressure.
- Found the satellite position of profound importance in the observed signature.
- Inward radial Poynting vector can be balanced by azimuthal Poynting vector.
- S_x can be used as an indicator of the driving phase.
- Can infer source region from Poynting vector.

Cold Plasma Equations

The equations then reduce to

$$
\begin{aligned}
\frac{\partial b_x}{\partial t} &= -k_z u_x, \\
\frac{\partial b_y}{\partial t} &= -k_z u_y, \\
\frac{\partial b_z}{\partial t} &= -\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}\right), \\
\frac{\partial u_x}{\partial t} &= \frac{1}{\rho}\left(k_z b_x - \frac{\partial b_z}{\partial x}\right), \\
\frac{\partial u_y}{\partial t} &= \frac{1}{\rho}\left(k_z b_y - \frac{\partial b_z}{\partial y}\right).
\end{aligned}
$$

which we solve with a Leapfrog-trapezoidal finite difference scheme of *Zalesak*, [1979].

Finite Difference Method

Can express equations in the form

$$\frac{\partial \mathbf{U}}{\partial t} = \mathbf{F}$$

where

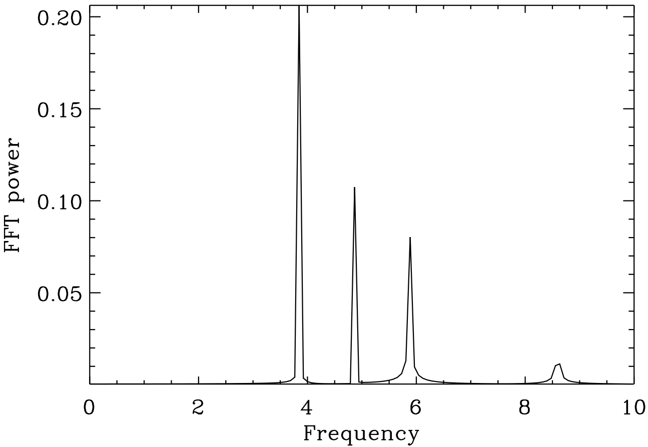
$$\mathbf{U} = \begin{pmatrix} u_x \\ u_y \\ b_x \\ b_y \\ b_z \end{pmatrix}, \qquad \mathbf{F} = \begin{pmatrix} \left(k_z b_x - b_{z,x}\right)/\rho \\ \left(k_z b_y - b_{z,y}\right)/\rho \\ -k_z u_x \\ -k_z u_y \\ -\left(u_{x,x} + u_{y,y}\right) \end{pmatrix}$$

Finite Difference Method

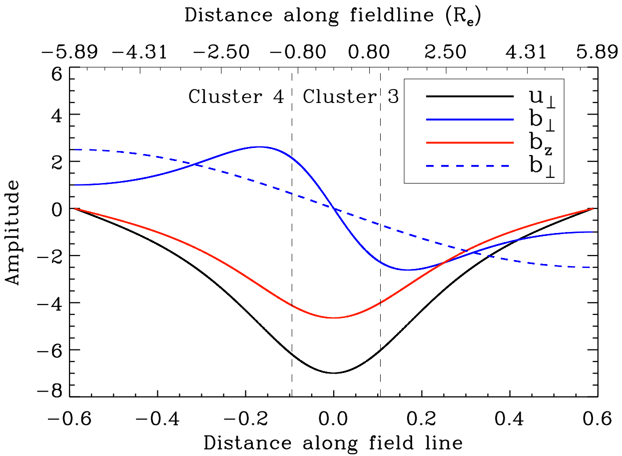
- Assuming we know $\mathbf{U}$ at times t and $t - \Delta t$, then the scheme is

$$\begin{aligned} \mathbf{U}^{\dagger} &= \mathbf{U}^{t-\Delta t} + 2\Delta t \mathbf{F}^{t} \\ \mathbf{F}^{*} &= \frac{1}{2}\left(\mathbf{F}^{t} + \mathbf{F}^{\dagger}\right), \\ \mathbf{U}^{t+\Delta t} &= \mathbf{U}^{t} + \Delta t \mathbf{F}^{*}. \end{aligned}$$

- Use centered finite differences to calculate the spatial derivatives.
- Scheme is second order accurate in time and space.



0.20
0.15
0.10
0.05
FFT power
0
2
4
6
8
10
Frequency


Distance along fieldline (R_e)
−5.89 −4.31 −2.50 −0.80 0.80 2.50 4.31 5.89
Cluster 4
Cluster 3
$u_\perp$
$b_\perp$
b_z
$b_\perp$
Amplitude
6 4 2 0 −2 −4 −6 −8
−0.6 −0.4 −0.2 0.0 0.2 0.4 0.6
Distance along field line